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General Response of Resistance Thermometers and Thermocouples in Gases at Low Pressures

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ALL types of thermometer used to measure gas temperature rely upon the thermal contact between the sensing element and the gas to achieve a good reponse. It is necessary that this thermal contact should predominate over the thermal contact between the element and its mountings if the sensor is not to indicate the temperature of the mountings. At low-gas pressures, the surface heat-transfer coefficient between the sensing element and the gas can be sufficiently small so that the sensor tends to adopt the temperature of the mountings rather than that of the gas. Heat-transfer coefficients from fine wires under these conditions have recently been discussed by Rajasooria and Brundrin.[1] In this Note, the general response of resistance wires and wire thermocouple junctions is considered in terms of the relation between the average wire temperature (T_{av}) (or junction temperature (T_c) for the thermocouple) and the temperature of the gas (T_f) and wire mountings (T_o). It is assumed that the mountings are sufficiently massive so that their temperature is not affected by the flow.

The thermal equilibrium of an unheated resistance wire is determined by the equation for heat conduction along the wire length (direction x), the local wire temperature (T) being given by

$$d^2T/dx^2 - (4h/kd)(T - T_f) = 0 \quad (1)$$

where h is the heat-transfer coefficient for the surface of the wire, of diameter d and thermal conductivity k. These parameters are assumed constant over the wire length. The equation is identical to that used by King[2] and many others subsequently, except that the heating term due to the current in the wire is omitted in this case. The solution to this equation is determined by the boundary conditions ($T = T_0$ at $x = \pm l$) and is given by

$$\theta = (T - T_o)/(T_f - T_o) = 1 - \cosh(a\eta)/\cosh(al) \quad (2)$$

where $a = (4h/kd)^{1/2}$ and $\eta = x/l$. The average wire temperature, obtained simply by integrating over the wire length ($-l < x < l$), is

$$\theta_{av} = (T_{av} - T_o)/(T_f - T_o) = 1 - \sinh(al)/[al\cosh(al)] \quad (3)$$

Figure 1 shows a set of dimensionless temperature profiles along the wire, the average temperature increasing and the profile becoming flatter as al becomes large and the wire temperature approaches the fluid temperature. Figure 2 shows the variation of average wire temperature, this being midway between support and fluid temperatures (i.e., $\theta_{av} = 0.5$) if $al = 1.94$.

For some typical resistance wires (Table 1) it may be seen that

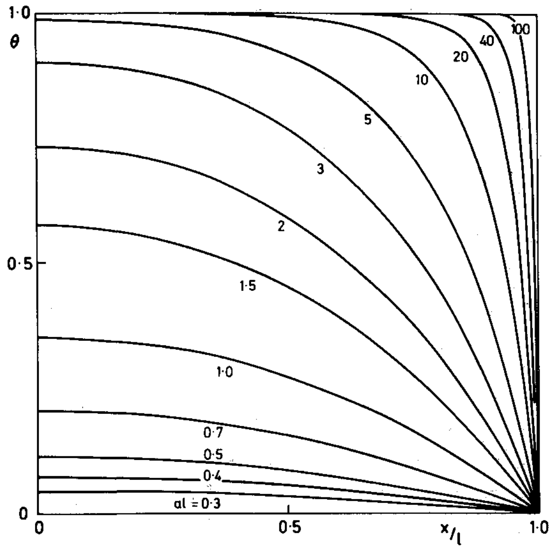

Fig. 1 Temperature distributions along resistance wires.

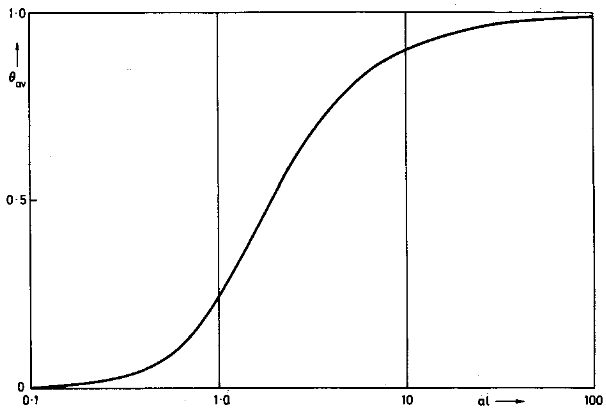

Fig. 2 Variation of average resistance wire temperature.

the heat transfer coefficient corresponding to a relatively low response to fluid temperature (e.g., $\theta_{av} = 0.1$) lies in the range of free molecular heat transfer at the wire surface. The heat transfer coefficient in a static gas is then given by (Kennard[3])

$$h = (C_v + R/2)\alpha p/(2\pi MRT_f)^{1/2} \quad (4)$$

where α is the surface thermal accommodation coefficient, M, C_v and R being the molecular weight, specific heat at constant volume and gas constant for the gas. Using this equation with $\alpha = 1$, values of the air pressure corresponding to $\theta_{av} = 0.1$ are also shown in Table 1. When the gas is moving at a significant Mach number, the analysis of Oppenheim[4] or Schaaf[5] may be used to determine the surface heat-transfer coefficient under free molecular conditions. For the limiting conditions of large M (approximately $M > 2$) the Stanton number becomes constant and the heat transfer coefficient is determined directly by the gas flux density. Flux density values corresponding to $\theta_{av} = 0.1$ are also shown in Table 1 for the typical wires considered.

Where a thermocouple is constructed of a bead junction supported by the two connecting wires, a similar analysis may be applied by using Eq. (1) for each support wire (with $T = T_o$ at $x = -l_1$ and $x = +l_2$). Constants $a_1 = (4h_1/k_1d_1)^{1/2}$ and $a_2 = (4h_2/k_2d_2)^{1/2}$ are introduced for each support wire and the equation for bead thermal equilibrium is also required to solve for the junction bead temperature T_c. That is, assuming the same heat-transfer coefficient h for all surfaces.

$$hA(T_c - T_f) = (\pi d_2^2 k_2/4)(dT_2/dx)_{x=0} - (\pi d_1^2 k_1/4)(dT_1/dx)_{x=0} \quad (5)$$

where A = area of bead surface at $x = 0$. This leads, together

Received October 26, 1971.
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Table 1 Lower limits for practicable operation of resistance wires and thermocouples (Values given are for $\theta_{av}, \theta_c = 0.1$)

Sensor details	h, w cm^{-2} °C^{-1}	Static air (p)		Flux density (ρv), ($M > 2$)
		p, torr	λ/d	(g cm^{-2} sec^{-1})
Platinum wire, $2l = 2$ mm, $d = 3 \times 10^{-3}$ mm	1.9×10^{-3}	0.12	68	2.1×10^{-3}
Tungsten wire, $2l = 2$ mm, $d = 5 \times 10^{-3}$ mm	6.7×10^{-3}	0.43	24	7.4×10^{-3}
Copper/constantan thermocouple, $l_1 = l_2 = 5$ mm, $d = 25 \times 10^{-3}$ mm				
(a) Negligible bead size ($\bar{A}_1 = \bar{A}_2 = 0$)	1.0×10^{-3}	0.066	32	1.2×10^{-3}
(b) Large bead size ($\bar{A}_1 = \bar{A}_2 = 10$)	9.6×10^{-5}	0.0061	25 (bead)	1.1×10^{-4}

with solutions from equation 1 for each support wire, to the result

$$\theta_c = (T_c - T_o)/(T_f - T_o) = [(\bar{A}_1 a_1 l_1 \sinh a_1 l_1)^{-1} + (\bar{A}_2 a_2 l_2 \sinh a_2 l_2)^{-1}]/[1 + (\bar{A}_1 a_1 l_1 \tanh a_1 l_1)^{-1} + (\bar{A}_2 a_2 l_2 \tanh a_2 l_2)^{-1}] \quad (6)$$

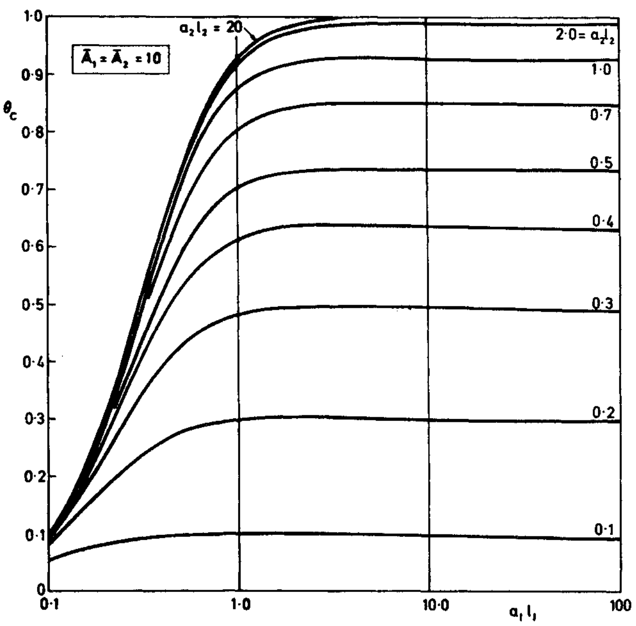

Fig. 3a Variation of thermocouple junction temperature (negligible bead size).

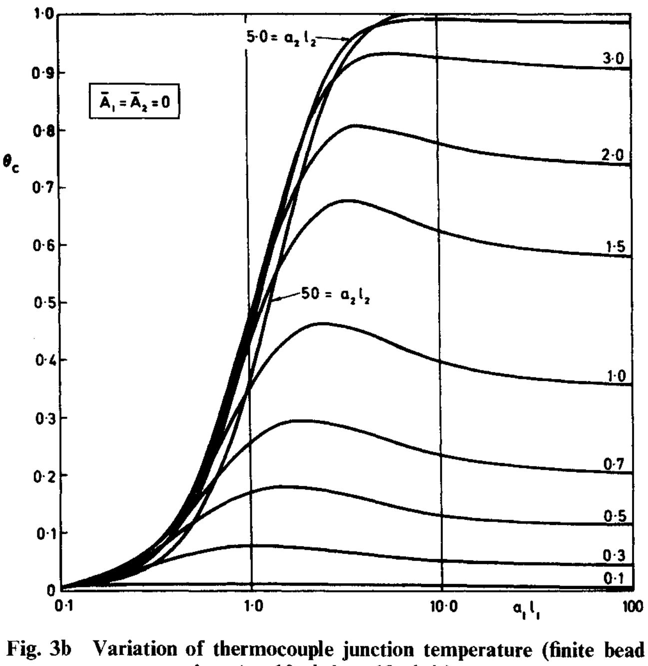

Fig. 3b Variation of thermocouple junction temperature (finite bead size, $A = 10\pi d_1 l_1 = 10\pi d_2 l_2$).

where $\bar{A}_1 = A/\pi d_1 l_1$ and $\bar{A}_2 = A/\pi d_2 l_2$. Solutions for the thermocouple response are shown in Fig. 3, assuming that the surface areas of the two support wires are equal (i.e., $\bar{A}_1 = \bar{A}_2$). It may be seen that values of both $a_1 l_1$ and $a_2 l_2$ in excess of approximately 5 are required if the thermocouple temperature is to approach the fluid temperature closely. Typical values for a copper-constantan junction supported on two 5mm wires in the gas are given in Table 1 for the cases of negligible bead size ($A_1, A_2 = 0$) as well as for the case of a relatively large bead ($\bar{A}_1 = \bar{A}_2 = 10$), in this case a value of $\theta_c = 0.1$ being assumed.

In conclusion, it may be seen that the solutions given provide a relatively simple means of estimating the response of resistance wire thermometers and thermocouples under conditions of marginal heat transfer between the sensor and gas. The results are of particular relevance to the measurement of gas temperatures at low pressures where significant temperature differences between gas and thermometer mountings exist.

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Wind-Tunnel Wall Interference Reduction by Streamwise Porosity Distribution

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THE interference on the flowfield about a model caused by wind-tunnel walls is well known as one of the sources that influences the accuracy of tunnel data.[1–4] After the ventilated

Received November 11, 1971; revision received November 29, 1971. The research reported herein was sponsored by the Arnold Engineering Development Center, Air Force Systems Command, under Contract F40600-72-C-0003 with ARO Inc. Major financial support was provided by the Air Force Flight Dynamics Laboratory under Air Force Project 1366. Project monitor was Maj. P. J. Butkewicz. Further reproduction is authorized to satisfy the needs of the U.S. Government.

Index categories: Aircraft and Component Wind Tunnel Testing; Subsonic and Transonic Flow.

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